

Chemical Engineering Analysis 1
Mid-term Exercise

ChE 735

Fall 2019

October 11, 2019

Mid-term exercise, submission deadline 10/20/2019, 5 PM. Strictly individual effort expected.

Let's think about the following scenario: A ship, fleeing from pirates, sails in a straight line parallel to the y -axis. The pirate ship, starting from the origin, keeps her bow pointed continuously at the pursued. Naturally, the pirate ship's speed is greater than that of the fleeing vessel. Where will the paths of the two ships intersect, i.e., where will capture of the fleeing ship take place? This is the classic curve of pursuit problem which appears to have been first considered in detail by Pierre Bouguer (a French hydrographer) in 1732.

The fleeing vessel starts from the x -axis at a position corresponding to $(x=a, y=0)$, and it sails perfectly parallel to the y -axis; the pursuing vessel starts from the origin $(x=0, y=0)$. The pursuit curve is governed by the ordinary differential equation,

$$1 + \left(\frac{dy}{dx}\right)^2 = k^2(a-x)^2 \left(\frac{d^2y}{dx^2}\right)^2$$

k is the ratio of the velocities, and obviously it must be greater than 1 (one). Capture can only occur when the pirate vessel attains the position $(x=a, y=y_{\text{capture}})$.

Part 1. Work problem 6.18 from the text, using three different values for k : 1.2, 1.5, and 1.7 (with $a=10$).

Part 2. This is a very specific variation of problem 6.19: Imagine that a surveillance aircraft (SR-71) flies directly over a missile-equipped anti-aircraft installation. The SR-71 flies at a constant speed of 2045 mph at a fixed altitude of 89,500 ft. A ground-to-air missile is fired and it rapidly attains its terminal speed which is sustained until the rocket engine runs out of fuel—this occurs at exactly 38 seconds. What is the threshold speed that the missile must have in order to effect an interception and precisely where does the interception take place? Note that if the missile's speed is 3600 mph for exactly 38 seconds, it will have travelled about 200,640 ft. At 2800 mph, the missile travels 156,053 ft in 38 seconds. Of course, once its fuel is exhausted its speed will fall off very quickly!