Advanced Transport Phenomena 1

ChE 862 **March 8, 2019** **Spring 2019**

Due: On or before April 5, 2019

Optional exercise: Use of the vorticity transport equation for modeling recirculating flow in a square cavity. The flow is driven by the motion of the upper planar surface. It slides across the cavity with constant velocity, V. The fluid is initially at rest and the vorticity created by the moving surface is transported throughout the cavity.

The principal equations we must deal with in this problem are vorticity transport and a Poisson-type equation relating the stream function to the vorticity distribution:

$$\frac{\partial \omega}{\partial t} + v_x \frac{\partial \omega}{\partial x} + v_y \frac{\partial \omega}{\partial y} = \nu \left[\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right] \quad \text{and} \quad -\omega = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}.$$

As we discussed in class, both of these equations can be solved by elementary methods; vorticity transport explicitly by forward marching in time, and the Poisson equation iteratively (I recommend SOR). The main problem is that we must use upwind differencing on the convective transport terms, e.g., $v_x \frac{\partial \omega}{\partial x}$ will require that we determine the direction of the flow in order to compute the gradient properly. One attractive feature of this problem is that we have a "fallback" position if the difficulties you encounter are too severe: We can assume that the Reynolds number is very low such that vorticity merely diffuses through the cavity. This is described in the text in section 3.11. Some additional detail appears in problem 8.18 in *Applied Mathematics for Science and Engineering* (John Wiley & Sons, 2014). If you decide to take this approach you may also want to look at the example in C. Y. Chow, *An Introduction to Computational Fluid Mechanics*, 1979. The square cavity is 5 cm on each side, the kinematic viscosity (ν) is 0.10 cm^2/s, and the velocity of the sliding plate (V) is 1 cm/s. An illustration of a flow with the same Reynolds number (but in a deep cavity) is provided below.