Advanced Transport Phenomena 1

ChE 862 **April 17, 2019** **Spring 2019**

Due: April 24, 2019

A fully developed flow of water occurs between parallel, planar surfaces. The upper wall (at y=B=2 cm) is maintained at a uniform temperature of 80 °C and the lower wall located at y=0 is insulated. The fluid properties are taken to be constant and the Reynolds number is 1800. The water enters the heated section with a uniform temperature of 2 °C. We need to determine the bulk fluid temperature and the Nusselt number as functions of distance from the entrance.

The lower surface is corresponds to y=0 and the upper surface to y=B. The laminar flow is fully developed and the velocity distribution is

$$v_x = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - By\right) .$$

Since Re=1800, $\langle v_x \rangle = 9 cm/s$, and the value for dp/dx is set accordingly. Heat transfer in this steady-state process is governed by

$$\rho C_p v_x \frac{\partial T}{\partial x} = k\left[\frac{\partial^2 T}{\partial y^2}\right];$$

you will notice immediately that conduction in the flow direction, $\frac{\partial^2 T}{\partial x^2}$, has been omitted. This is generally permissible as long as the product, $RePr$, is greater than about 100. In our case, $RePr$ is about 12,600 so the simplification is appropriate. We want to find the bulk fluid (or mixing cup) temperature and the Nusselt number as functions of distance from the entrance (x). We will define the Nusselt number as $\frac{hB}{k}$ and we note that it must be determined by establishing a Robin's type boundary condition at the upper surface:

$$-k\frac{\partial T}{\partial y}_{y=B} = h\left(T_m - T_{y=B}\right).$$

Because the fluid velocity varies with y-position, the bulk fluid temperature *must* be determined by integration: $T_m = \frac{1}{WB\langle v_x \rangle}\int_0^B W v_x(y) T(y) dy$.