January 2019

Advanced Transport Phenomena 1

ChE 862 Spring 2019

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Course content:

1. **The equations of fluid motion, boundary conditions and nonlinearity.**
2. **Forces exerted by fluids and the stress tensor.**
3. **Couette flows, Poiseuille flows and the laminar boundary layer.**
4. **Stability and introduction to the theory of small disturbances.**
5. **Transition to turbulence and review of turbulence fundamentals.**
6. **Molecular heat and mass transfer (conduction and diffusion).**
7. **Heat and mass transfer with laminar fluid motion.**

Required text: ***Transport Phenomena: An Introduction to Advanced Topics*****, by Larry A. Glasgow (John Wiley & Sons, ISBN 978-0-470-38174-8). We will also refer occasionally to several useful texts and monographs including:**

Transport Phenomena**, Bird, Stewart, & Lightfoot**
Boundary-Layer Theory**, Schlichting**
An Introduction to Turbulence and its Measurement**, Bradshaw**

Grading: We will have about 15 graded assignments. The course normally includes two examinations (mid-term and final) although the format of these tests varies from year to year.

Objectives of course:
Acquaint student with important topics in advanced transport phenomena.
Develop physical understanding of, and the ability to apply, the principles discussed.
Familiarize student with classic and current literature in the field.

Philosophy:
The success of graduate-level coursework depends almost entirely upon effort expended by the student. We will work a wide variety of problems, some simple and some quite difficult; many of the problems will be interesting and hopefully, all of them will provide opportunities for learning. We would also like to encourage classroom discussion-- it is almost always a more effective learning device than the formal lecture. ***Please*** **do <u>not</u> hesitate to interject. We want to encourage a participatory classroom.**

Mandated Statements:

1. **Statement Regarding Academic Honesty**

   Kansas State University has an Honor System based on personal integrity, which is presumed to be sufficient assurance that, in academic matters, one's work is performed honestly and without unauthorized assistance. Undergraduate and graduate students, by registration, acknowledge the jurisdiction of the Honor System. The policies and procedures of the Honor System apply to all full and part-time students enrolled in undergraduate and graduate courses on-campus, off-campus, and via distance learning. The honor system website can be reached via the following URL: www.ksu.edu/honor . A component vital to the Honor System is the inclusion of the Honor Pledge which applies to all assignments, examinations, or other course work undertaken by students. The Honor Pledge is implied, whether or not it is stated: "On my honor, as a student, I have neither given nor received unauthorized aid on this academic work." A grade of XF can result from a breach of academic honesty. The F indicates failure in the course; the X indicates the reason is an Honor Pledge violation.

2. **Statements for Academic Accommodations for Students with Disabilities***

   "Any student with a disability who needs a classroom accommodation, access to technology or other academic assistance in this course should contact Disability Support Services (dss@k-state.edu) and/or the instructor. DSS serves students with a wide range of disabilities including, but not limited to, physical disabilities, sensory impairments, learning disabilities, attention deficit disorder, depression, and anxiety."

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   * Faculty members who have a student with a disability in their class will want to contact the office of Disability Support Services (DSS) office on our campus. DSS will help faculty provide academic accommodations for students with documented disabilities. Students with disabilities include those having mobility impairments, visual and hearing impairments, chronic health conditions, learning disabilities and attention deficit disorder. DSS is located in Holton Hall 202. The Director, Andrea Blair, can be contacted at 532-6441.

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3. **Statement Defining Expectations for Classroom Conduct**

   All student activities in the University, including this course, are governed by the Student Judicial Conduct Code as outlined in the Student Governing Association By Laws, Article VI, Section 3, number 2. Students who engage in behavior that disrupts the learning environment may be asked to leave the class.

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An introductory example that establishes a framework for our early-semester discussions:

Consider the elementary, nonlinear difference (logistic) equation:

$$X_{n+1} = \alpha X_n (1 - X_n) \qquad (1)$$

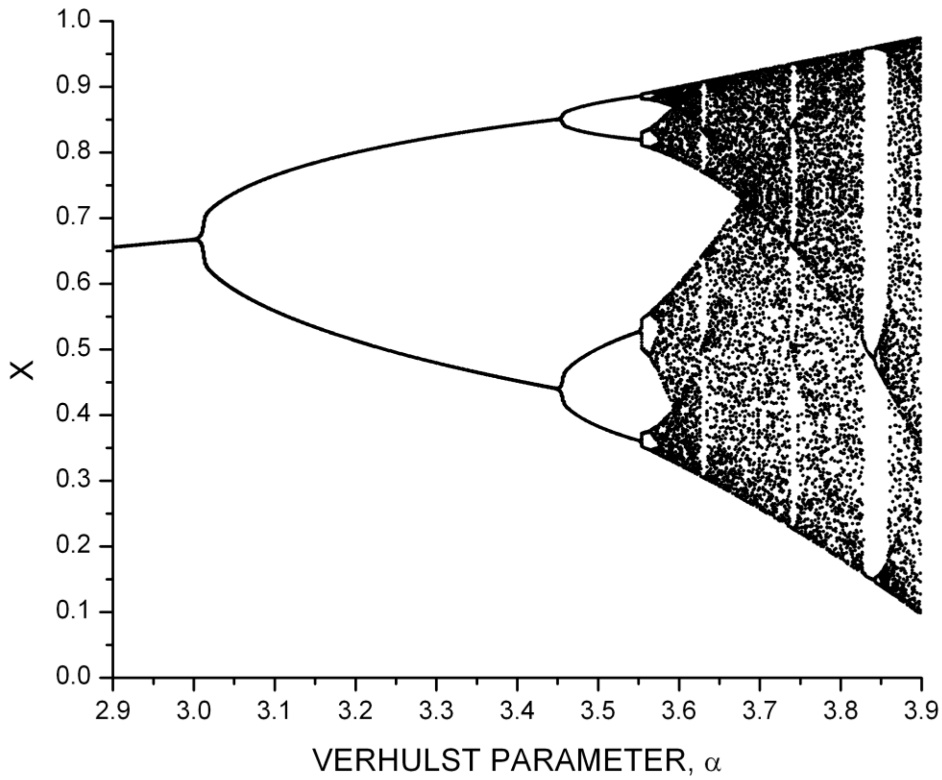
This equation is a modification of the Malthusian population model (Thomas Robert Malthus, 1766-1834) in which the leading constant can be taken as the growth rate. Problems of population were extensively studied by Verhulst in the mid-19th century; the Verhulst (differential) equation is:

$$\frac{dX}{dt} = AX - BX^2 \qquad (2)$$

Note the obvious connection between the Verhulst and logistic equations. You may also see that (2) is a Riccati equation; although it is nonlinear, its solution poses no serious problem for us today. That wasn't always the case. These (now) elementary considerations led Volterra and Lotka to examine two populations in conflict, say foxes and rabbits. Many rabbits and the foxes eat well and prosper. Few rabbits and the foxes starve. Such models must involve simultaneous, nonlinear differential equations. In phase space, we might expect limit cycle behavior from such models. After just a moment's reflection you will see that this is not entirely realistic. In biological systems extinction is always a possibility. Furthermore, greed, hunger, sex, and moral restraint are probably not deterministic processes! Let's turn our attention back to eq. (1); let the growth rate constant, α, assume a numerical value of, say, 2.75. Now let the population variable (x) be 0.045. We apply (1) repetitively and obtain the sequence:

n	x
1	0.045
2	0.118181
3	0.28659
4	0.67684
5	0.601499
6	0.65917
7	0.61783
8	0.64932
9	0.62618
10	0.64371
....	
19	0.63580
....	
140	0.636364

Interesting, but expected. Now suppose we allow the growth rate constant to increase--we calculate about ten x's and then set $\alpha=\alpha+0.02$. Repeat. Again. If we do this enough, and figure out how to plot the results, we'll generate a *bifurcation* diagram. That such complex behavior could emerge from such a simple nonlinear algebraic equation is stunning. You may want to try to find the first few critical values of α.



You can see above how certain values of the parameter, α, lead to branching (bifurcation). When this happens successively as illustrated here, we quickly arrive at an unexpected level of complexity! Note that there are regions with chaotic behavior and narrow bands (or windows) of periodicity. To learn more about this kind of behavior, see Baker and Gollub (*Chaotic Dynamics, An Introduction,* Cambridge University Press, 1990). The phenomenon illustrated above has some fascinating (and perhaps frightening) implications with respect to fluid mechanics. Consider the first couple of (inertial) terms of the x-component of the Navier-Stokes equation:

$$\rho\left(\frac{\partial V_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}+........\right) \qquad \textbf{(3)}$$

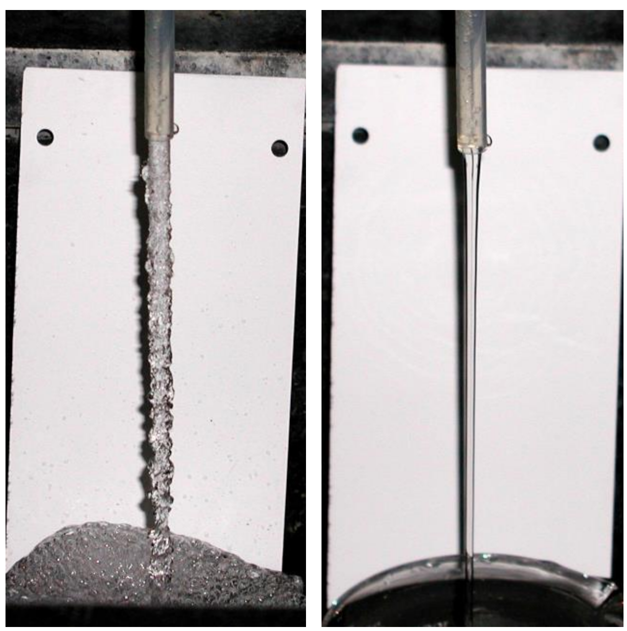
Do you see any possible connection? You might try to write appropriate difference approximations for these terms; let the *i* subscript represent position and the *j* subscript represent time. Use a simple first-order forward difference approximation for the time derivative as though you planned to solve the hypothetical problem explicitly. The similarity between your result and the logistic equation should be apparent! One possibility:

$$\rho\left(\frac{V_{i,j+1}-V_{i,j}}{\Delta t}+V_{i,j}\frac{V_{i,j}-V_{i-1,j}}{\Delta x}+....\right) \qquad \mathbf{(4)}$$

Look carefully at the convective term. Hmmm..... This will be important to us later.
The significance of the preceding discussion relates to our efforts to solve the Navier-Stokes equation. Since the middle of the 19th century when such efforts began, only about 75 to 80 exact solutions have been found. Many of these fall into three groups: pressure-driven (Poiseuille) duct flows, shear-driven (Couette) flows, and creeping fluid motions where the viscous forces are dominant. This last case, of course, is equivalent to stating that the Reynolds number is extremely small. Recall that the Reynolds number is a comparison of inertial and viscous forces. This in turn brings up an interesting point. For Hagen-Poiseuille flow (laminar flow in a cylindrical tube), Re cannot be regarded as a natural parameter. Why not? What about Couette flow between concentric cylinders?

The nonlinear partial differential equations that are descriptive of more complicated flows have an unfortunate characteristic: They are extremely sensitive to initial conditions. This feature, and its implications for the analysis of fluid flow <u>began</u> to be widely appreciated following the dissemination of Edward Lorenz's work in 1963. It means, for example, that truly long-range weather prediction may remain unrealizable. Consider the flash photographs shown below of water flowing from a laboratory tap--note the surface of each of the jets.

If we took a hundred or even a thousand such photographs, would we ever see the exact same conformation twice for the turbulent jet?



You can explore and appreciate this difficulty yourself with any number of simple experiments. Here's one that requires a cardboard box with a circular hole in one side and just a little dry ice; the picture immediately below shows a kinetic "sculpture" (vortex ring generators) in downtown Louisville, KY.

You can generate your own vortices by impulsively striking the sides of your box. Observe the shape, translation, and decay of the ring. Although the Reynolds number for this flow is large, the vortex ring is surprisingly stable—it can travel large distances and remain intact. Why? Will two identical events ever occur?

<u>First assignment, due Friday, February 1, 2019</u>: Read Chapter 1 and complete the example problem started in class. Also, work problem 1A.