Advanced Transport Phenomena 1

ChE 862 **April 3, 2019** **Spring 2019**

Due: April 10, 2019

A heated spherical solid is placed in a well-stirred calorimeter filled with cold water. We are interested in the temperature of the water as a function of time, and whether or not this arrangement could be used to estimate the thermal diffusivity of a solid sphere of unknown material. An analytic solution for this scenario is given by Carslaw and Jaeger in Section 9.6; we will solve the problem numerically however.

A solid granite sphere with R=3 cm is heated to a uniform temperature of 90 °C, then placed in a well-stirred calorimeter filled with cold water at 0 °C. The properties of granite (all cgs) are: ρ=2.6, C_p=0.21, k=0.006, and α=0.011. The calorimeter contains 1850 g of water. The governing equation for conduction in the interior of the sphere is:

$$\rho C_p \frac{\partial T}{\partial t} = k\left[\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right].$$

Of course we have symmetry at the center of the sphere but the really interesting boundary condition is the one at the surface:

$$-4\pi R^2 k \left.\frac{\partial T}{\partial r}\right|_{r=R} = M_f C_{pf} \frac{\partial T_f}{\partial t}.$$

That is, the heat lost at the surface of the sphere by conduction is gained by the cold water (M_f =1850 g). Since the water is well-stirred, the temperature at the surface of the sphere and the temperature of the water should be virtually identical. Prepare a graph of your numerical results in the form: $(k_p + 1)T(r = R)/90$ as a function of $\alpha t / R^2$. The parameter k_p is the ratio of the heat capacities of the fluid and the solid granite sphere:

$$k_p = \frac{3M_f C_{pf}}{4\pi R^3 \rho C_p}.$$

Finally, describe a strategy that would allow you to use this experimental arrangement to estimate the thermal diffusivity of some unknown spherical solid.